Integration / Integralrechnung

Differenzialrechnung

Integral

Riemannsummen

Aufleiten — Ableiten

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Tangentensteigung

Kurvendiskussion

Scheitelpunktform
einer Parabel / quadratische Funktion

$$\left(x^3 \cdot \sqrt{4x+1}\right)' = \left(x^3\right)' \cdot \sqrt{4x+1} + x^3 \left(\boxed{\sqrt{4x+1}}\right)' \quad *$$

$$= 3x^2 \cdot \sqrt{4x+1} + x^3 \frac{2}{\sqrt{4x+1}} = 3x^2\sqrt{4x+1} + \frac{2x^3}{\sqrt{4x+1}}$$

$$* \quad \boxed{(4x+1)^{\frac{1}{2}}} = \underbrace{\frac{1}{2}(4x+1)^{-\frac{1}{2}}}_{\text{Äussere Ableitung}} \cdot \underbrace{4}_{\text{Innere Ableitung}} = 2\frac{1}{\sqrt{4x+1}}$$

Ketten-
regel

$$x^{-3} = \frac{1}{x^3}$$

$$v(u) = \sqrt{u} \qquad v'(u) = \left(u^{\frac{1}{2}}\right) = \frac{1}{2}u^{-\frac{1}{2}} = \boxed{\frac{1}{2\sqrt{u}}}$$

$$u(x) = 4x+1 \qquad u'(x) = 4x^0 + 1 \cdot 0 \cdot x^{-1} = 4 + 0 = 4$$

$$v'(x) = \frac{1}{2\sqrt{u}} \cdot 4 = \frac{4}{2\sqrt{4x+1}} = \frac{2}{\sqrt{4x+1}}$$

$$\left(4^{e^x}\right)' = \ln(4)\cdot 4^u \cdot e^x = \ln(4)\cdot 4^{e^x}\cdot e^x$$

$v(u) = 4^u$ $\qquad v'(u) = \ln(4)\cdot 4^u$

$u(x) = e^x$ $\qquad u'(x) = e^x$

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1)

Nullstellen: $f(x) = 0$ $\qquad \mathbb{D} = \mathbb{R} \setminus \{-2, 2\}$

$x^2 - 4 = 0 \Rightarrow 2, -2$ (Definitionslücken)

$$\frac{2x^2-2}{x^2-4} = 0 \rightarrow 2x^2 - 2 = 0$$

$$2x^2 = 2 \rightarrow \underline{\underline{x = \pm 1}}$$

Extrema: $f'(x) = 0$

Quotientenregel: $\left(\frac{f_1(x)}{f_2(x)}\right)' = \frac{f_1'(x)\cdot f_2(x) - f_2'(x)\cdot f_1(x)}{f_2(x)^2}$

$f_1(x)' = (2x^2-2)' = 4x$

$f_2(x)' = (x^2-4)' = 2x$ $\Big\}$ einsetzen

$$\frac{4x\cdot(x^2-4) - 2x\cdot(2x^2-2)}{(x^2-4)^2} = \frac{-12x}{(x^2-4)^2} = f'(x)$$

$$\frac{-12x}{(x^2-4)^2} = 0 \longrightarrow x = 0$$

Maxima oder Minima?

$$\frac{-12x}{(x^2-4)^2} \rightarrow \text{Quotientenregel}$$

$f_1(x)'' = -12$

$f_2(x)'' \rightarrow$ Kettenregel: $(v'(u)\cdot u'(x))$

$2\cdot(x^2-4)$ $\qquad \Rightarrow 2\cdot(x^2-4)\cdot 2x = 4x(x^2-4)$

$2x$

→ Quotientenregel: $\frac{-12\cdot(x^2-4)^2-4x(x^2-4)\cdot(-12x)}{(x^2-4)^4}$

→ $x=0$ einsetzen

→ $f(0) = -\frac{3}{4}$ → Maximum

Wendepunkte: $f''(x)=0$

→ false → gibt keinen

Definitionsbereich

$f(x) = x\cdot\sqrt{a-x^2}$ $\qquad x = 2$

$f(x)' = 1\cdot\sqrt{a-x^2} + x\cdot\left[\underbrace{(a-x^2)^{1/2}}_{b}\right]'$

$b: \frac{1}{2}\cdot(a-x^2)^{-1/2}\cdot 2\cdot(-x)$

$f'(x) = 1\cdot\sqrt{a-x^2} + x\cdot 1/2\cdot(a-x^2)^{-1/2}\cdot 2\cdot(-x)$

$f'(2) = \sqrt{a-4} + \frac{2}{2}\cdot(a-4)^{-1/2}\cdot 2\cdot(-2) = 0$

nach a auflösen → solve

$\Rightarrow \underline{a = 8}$

$\sqrt{a-4} = \frac{4}{\sqrt{a-4}}$ $\quad$ | hoch 2

$$a - 4 = \frac{16}{a-4} \quad \Big| \cdot (a-4)$$

$$a^2 - 8a + 16 = 16 \quad \Big| -16 + 8a$$

$$a^2 = 8a \quad \Big| : a$$

$$a = 8$$

b) $f(x) = \ln(x)$ Tangentengleichung: $y = mx + b$

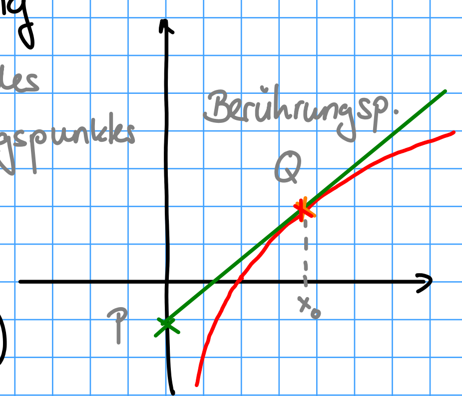
$P_0(0/-2)$ auf der Tangente $\Rightarrow -2 = m \cdot 0 + b$

$\Rightarrow b = -2$

<u>Zu m:</u> Steigung der Tangente = Ableitung

$m = f'(x) = (\ln(x))' \Rightarrow m = \frac{1}{x_0}$ x-Wert des Berührungspunktes

$y_0 = \ln(x_0)$



$y = \frac{1}{x} \cdot x - 2 = y = -1$ f

Nicht dieselben x.

$Q(x_0 / \ln(x_0))$

Einsetzen

$$\ln(x_0) = \frac{1}{x_0} x_0 - 2$$

$$\ln(x_0) = -1$$

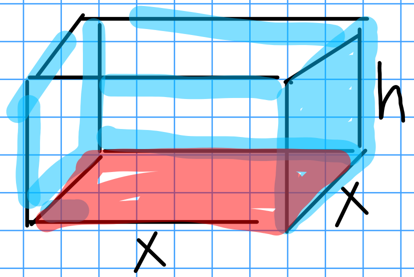
$\log_b(a) = x \Leftrightarrow b^x = a$

$$e^{-1} = x_0$$

$$m = \frac{1}{e^{-1}} = e$$

$$\underline{\underline{y = e \cdot x - 2}}$$

3)



$1L = 1\,dm^3 = 1000\,cm^3$

① $4xh + x^2$ mögl. klein

② $x^2 h = 1000 \rightarrow h = \dfrac{1000}{x^2}$

③ $\underbrace{x^2}_{2x} + \underbrace{4x\dfrac{1000}{x^2}}$

$\hookrightarrow \dfrac{4\cancel{x}\cdot 1000}{x^{\cancel{2}}} = \dfrac{4000}{x}$

$\left(\dfrac{4000}{x}\right)' = \left(4000 \cdot x^{-1}\right)' = -4000 \cdot x^{-2}$

$= -\dfrac{4000}{x^2}$

④ $f'(x) = 2x + \left(-\dfrac{4000}{x^2}\right) = 0$

$2x = \dfrac{4000}{x^2}$

$2x \cdot x^2 = 4000 \longrightarrow x = 12.59\,cm$

$x^2 \cdot h = 1000 \rightarrow h = 6.29\,cm$

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4 a) $f(x) = 0$ und $f'(x) = 0$

$f(x) = x^3 + 6x^2 + tx = 0 \quad |:x \quad \rightarrow x^2 + 6x + t = 0$

$f'(x) = 3x^2 + 12x + t = 0$

$3x^2 + 12x + t - x^2 - 6x - t = 0$

$2x^2 + 6x = 0$

$2x(x+3) = 0 \qquad x = 0$ oder -3

$3(-3)^2 + 12 \cdot (-3) + t = 0$

$t = 9$

b) $3x^2 + 12x + t \neq 0$

$$\frac{-12 \pm \sqrt{144 - 12t}}{6}$$

$144 - 12t < 0$

$t > 12$